DARKENBAYEVA GULSIM

ASYMPTOTIC THEORY OF REGRESSIONS WITH ASYMPTOTICALLY COLLINEAR REGRESSORS

ABSTRACT

of a dissertation is submitted in fulfilment of the requirements
for the degree of Doctor of Philosophy (PhD) on specialty
6D060100 –Mathematics

Topicality of the research theme. Consider a model

$$y_t = \alpha + \beta L(t) + u_t,\ t = 1,...,n,$$

where L is a positive, measurable on $[A,\infty)$, $A > 0$, and

$$\lim_{x\to\infty} \frac{L(rx)}{L(x)} = 1 \text{ for any } r > 0$$

function, or, shortly, L is a slowly varying function (SV). For the case when the errors $\{u_t\}$ are stationary, Phillips (Phillips P.C.B. Regression with slowly varying regressors and nonlinear trends // Econometric Theory. – 2007. – Vol. 23 (4). – P. 557–614) obtained the asymptotic distribution of the OLS estimators $\hat{\alpha}$ and $\hat{\beta}$.

We consider integrated errors

$$u_t = \rho u_{t-1} + v_t,\ t = 2,...,n,$$

where $\rho = 1$ under the null hypothesis and $\{v_t\}$ is a non-causal linear process

$$v_t = \sum_{i\in Z} c_i e_{t-i}\ .$$

Integrated errors and non-causal linear processes have many applications in statistics and econometrics. Results presented in this work can be used in derivation the limiting distribution of the unit root test statistic for our main regression model. Statement of this problem you can read in work of Uematsu (Uematsu Y. Regression with a Slowly Varying Regressor in the Presence of a Unit Root // Global COE Hi-Stat Discussion Paper Series

gd11-209, Institute of Economic Research, Hitotsubashi University. – 2011. – 29 p.). This problem is open at present time.

Also, as one can see we restrict our attention to models with deterministic regressors. Models with such regressors have many applications, see (Robinson P.M. Log-periodogram regression of time series with long range dependence // Annals of Statistics. – 1995. – Vol. 23 – P. 1048-1072; Hurvich C.M., Deo R., Brodsky J. The mean squared error of Geweke and Porter-Hudakís estimator of the memory parameter of a long-memory time series // Journal of Time Series Analysis. – 1998. – Vol.19 – P.19-46; Phillips P.C.B. Discrete Fourier Transforms of Fractional Processes. – Yale University, mimeographed. – 1999. – 59 p.; Wu C.F. Asymptotic theory of nonlinear least squares estimation // Annals of Statistics. – 1981. – Vol. 9 – P. 501-513; Barro R., Sala-i-Martin X. Economic Growth. – Cambridge, Massachusetts: The MIT Press. – 2004. – 160 p.; Phillips P.C.B., Sul D. Transition modeling and econometric convergence tests // Econometrica. – 2007. -Vol. 75(6) – P. 1771-1855; Phillips P.C.B., Sul D. Economic transition and growth // Journal of Applied Econometrics – 2009. – Vol. 27 – P. 1153-1185).

Another application of this research is to address the problem of early detection of bubbles. This is a macroeconomic problem that has direct implications for monetary and fiscal policies. A school headed by P. Phillips has provided a decisive component of the statistical procedure, see (Phillips P.C.B., Shi Shu-Ping, Yu J. Testing for Multiple Bubbles: Limit Theory of Real Time Detectors // International Economic Review. – 2015. – Vol. 56 – P. 1079-1134; Phillips Testing for Multiple Bubbles: Historical Episodes of Exuberance and Collapse in the S&P 500 // International Economic Review. – 2015. – Vol. 56 – P. 1043-1078; Phillips P.C.B. Exploring the Mysteries of Trends and Bubbles. // Australia's Economy and its International Context: The Joseph Fisher Lectures. - Adelaide: University of Adelaide Press, 2012. – Vol. II (1956–2012), Ch. 54. – P. 599–616; Phillips P.C.B., Yu J. Dating the Timeline of Financial Bubbles during the Subprime Crisis // Quantitative Economics. – 2010. – Vol. 1 – P. 455-491).

The aims and objectives of the study. The work is devoted to studying:

1) central limit theorems for quadratic forms of linear processes $\{v_t\}$;

2) add a couple of new sequences to the list of L_p-approximable sequences contained in Mynbaev (Mynbaev K. L_p-approximable sequences of vectors and limit distribution of quadratic forms of random variables // Advances in Applied Mathematics – 2001. – Vol. 26 (4). – P. 302–329.);

3) prove Uematsu's result (Uematsu Y. Regression with a Slowly Varying Regressor in the Presence of a Unit Root // Global COE Hi-Stat Discussion Paper Series gd11-209, Institute of Economic Research, Hitotsubashi University. – 2011. – 29 p.) on the asymptotic distribution of $\hat{\alpha}$ and $\hat{\beta}$ under less restrictive conditions.

The main provisions for the defense of the dissertation:

1) Obtained convergence of some quadratic forms used in regression analysis.

2) Obtained central limit theorems for linear and quadratic forms.

3) Added a couple of sequences to the list of L_p-approximable sequences contained in Mynbaev (16 Mynbaev K. -approximable sequences of vectors and limit distribution of quadratic forms of random variables // Advances in Applied Mathematics – 2001. – Vol. 26 (4). – P. 302–329.).

4) Proved Uematsu's result on the asymptotic distribution of OLS estimations $\hat{\alpha}$ and $\hat{\beta}$ under less restrictive conditions.

5) Done Monte-Carlo simulations for the asymptotic distribution of OLS estimations $\hat{\alpha}$ and $\hat{\beta}$.

The objects of research regression with slowly varying regressors, regression with asymptotically collinear regressors, non-causal linear processes, quadratic forms, central limit theorems.

The research subjects L_p-approximable sequences, quadratic forms of linear processes, central limit theorems.

Research methods L_p-approximation method of Mynbaev (Mynbaev K. L_p-approximable sequences of vectors and limit distribution of quadratic forms of random variables // Advances in Applied Mathematics – 2001. – Vol. 26 (4). – P. 302–329), central limit theorems.

Novelty of the dissertation research is that the main model with a slowly varying (SV) regressor in the presence of a unit root, also regression model has integrated errors $u_t = \rho u_{t-1} + v_t$, $t = 2,...,n$, and $\{v_t\}$ is a non-causal linear process.

Results in Section 3 generalize to the non-causal linear processes some statements from Phillips and Solo (Phillips P.C.B., Solo V. Asymptotics for linear processes // The Annals of Statistics. – 1992. – Vol. 20(2) – P. 971-1001) and Tanaka (Tanaka K. Time Series Analysis: Nonstationary and Noninvertible Distribution Theory. 1st ed. – Wiley and Sons, 1996. -623 p.).

Result in Section 4 extends Mynbaev's theorems (Mynbaev K. L_p-approximable sequences of vectors and limit distribution of quadratic forms of random variables // Advances in Applied Mathematics – 2001. – Vol. 26 (4). – P. 302–329) on convergence of quadratic forms to the case of asymmetric kernels.

Section 5 considers a couple of new L_p-approximable sequences.

The study of the main model with integrated errors gave us the asymptotic distribution of $\hat{\alpha}$ and $\hat{\beta}$. Uematsu characterized convergence in distribution of $\hat{\alpha}$ and $\hat{\beta}$, and it turned out to be very different from Phillips had with stable errors. In Section 6 we prove Uematsu's result on the asymptotic distribution of $\hat{\alpha}$ and $\hat{\beta}$ under less restrictive conditions.

Theoretical and practical significance of the research. This research constitute step in solving problem about unit root test. Also, has attracted number of applications in Econometrics and Statistics.

Connection of the dissertation thesis with the other scientific research works. The dissertation work was implemented within the scientific projects of the program of grant financing of fundamental researches in the areas of natural sciences of the Ministry of education and science of the Republic of Kazakhstan “Prediction of rare events and spatial effects in financial and commodity markets” (2015-2017 years, № 4084/GF4) and “Estimation of discontinuous densities and distribution functions in relation to applications in economics, finance and insurance” (2018-2020 years, AP05130154).

The work approbation. Results of the work were presented and discussed at the following conferences

- Darkenbayeva G. Регрессий с медленно меняющимися регрессорами // “Actual problems of pure and applied mathematics”, Almaty, 2015.
- Darkenbayeva G. Convergence of some quadratic forms used in regression analysis // Second International Conference on Statistical Distributions and Applications ICOSDA. – Niagara Falls, Canada, 2016.
- Darkenbayeva G. Сходимость некоторых видов квадратичных форм, возникающих в регрессионном анализе // XIII International scientific conference of students and young scientists “Lomonosov-2017”, Astana, 2017.

and seminars

- the city scientific seminar "Differential operators and their applications", Almaty, 2017;
- scientific seminar of Institute of mathematics, physics and informatics, Almaty, 2017;
- the city scientific seminar "Differential operators and their applications", Almaty, 2019.

Results of this dissertation were discussed with probability, statistics and econometrics specialist Carlos Brunet Martins-Filho during the scientific training in University of Colorado at Boulder, Colorado, USA, 2016.

Publications. Based on results of the dissertation 7 works were published: 5 journal articles (1 in Scopus indexed Journal and 4 in journals recommended by the Committee for Control in Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan):

1. Mynbaev K., Darkenbayeva G. Weak convergence of linear and quadratic forms and related statements on Lp-approximability // Journal of Mathematical Analysis and Applications. – 2019. – Vol. 473 – P.1305-1319.
2. Mynbaev K., Darkenbayeva G. The asymptotic distribution of the OLS estimator of the regression with slowly varying regressor // Математический журнал. – 2015. - Т. 15, №2. – С. 80-98.
3. Mynbaev K.,Darkenbayeva G. Convergence of some quadratic forms used in regression analysis // Математический журнал. – 2016. - Т. 16, №3. – С. 156-165.
4. Mynbaev K., Darkenbayeva G. Monte-Carlo study for OLS estimators for regression with slowly varying regressor // Математический журнал. – 2017. - Т. 17, №3. – С. 84-90.

5. Mynbaev K., Darkenbayeva G. Analyzing variance in central limit theorems // Kazakh Mathematical Journal. – 2019. - V. 19, №3. – P. 30-40.

and 2 in proceedings of international scientific conferences:

1. Darkenbayeva G. Регрессий с медленно меняющимися регрессорами // Тезисы докладов международной конференции «Актуальные проблемы математики и математического моделирования», посвященный 50-летию создания Института математики и механики. – Алматы, 2015. – С.24.

2. Darkenbayeva G. Сходимость некоторых видов квадратичных форм, возникающих в регрессионном анализе // Тезисы докладов XIII Международной конференции студентов, магистрантов и молодых ученых «Ломоносов - 2017». – Астана, 2017. – С. 9.

The reliability and validity of the scientific provisions, conclusions and results of the dissertation is confirmed by the publication of the results in the journal that have a non-zero impact factor.

Assessment of the completeness of the aims of the work. All results are new and based on our own methods and tools. In this work we have:

1) obtained convergence of some quadratic forms used in regression analysis.

2) obtained central limit theorems for linear and quadratic forms.

3) added a couple of sequences to the list of L_p-approximable sequences contained in Mynbaev.

4) proved Uematsu's result on the asymptotic distribution of OLS estimations $\hat{\alpha}$ and $\hat{\beta}$ under less restrictive conditions.

5) done Monte-Carlo simulations for the asymptotic distribution of OLS estimations $\hat{\alpha}$ and $\hat{\beta}$.

Therefore, the work objectives were completed.

Volume and structure of the dissertation. The work includes the title page, contents, normative references, definitions, notations and abbreviations, introduction, 7 sections, conclusion and references. Total volume of dissertation is 72 pages, the work contains 5 illustrations, 1 table and 42 literature references.

Main content of the dissertation. The introduction includes actuality of the research theme, aims and objectives, the main provisions for the defense of the dissertation, the research object and subject, methods, novelty and theoretical and practical significance of the research, connection of the dissertation thesis with the other scientific research works, the work approbation, author's publications, and volume, structure and content of the dissertation thesis.

The first section contains a more detailed introduction to the work.

The second section gives preliminary results like useful lemmas, theorems and assumptions which will be used in dissertation work.

In Section 3 we consider convergence in distribution of two quadratic forms arising in unit root tests for a regression with slowly varying regressor.

The fourth section gives central limit theorems for linear and quadratic forms with proofs.

The fifth section is about slow variation and L_p-approximality. Here we add a couple of new sequences to the list of L_p-approximable sequences contained in Mynbaev with proofs.

The sixth section contains the proof of the asymptotic distribution of the OLS estimators $\hat{\alpha}$ and $\hat{\beta}$.

The seventh section contains the Monte-Carlo simulations for the OLS estimators $\hat{\alpha}$ and $\hat{\beta}$.

The conclusion lists and generalizes the main results obtained during the implementation of the dissertation thesis.